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★ Normal Subgroup :-

1- Let G be a group. H is Subgroup of G. then H is called Normal Subgroup of G Iff :-

$$ghg^{-1} \in H \quad | \forall h \in H \quad | \forall g \in G$$

2) G – Group	$G \to$ Group
$R \to$ Normal	$H \to$ Normal
$g \in G$	$g \in G$
$n \in R$	$h \in H$
$gng^{-1} \in R$	$ghg^{-1} \in H$

★ **Theorem** → Every Subgroup of Abalian group is Normal

Proff → Let G be a Abelian Group

Let H be a Subgroup of G

We have to show that :-

H is Normal Subgroup of G

Step 2 → Let, $h \in H$, $g \in G$

$$ghg^{-1} = hg\,g^{-1} \qquad [g^{-1} = \tfrac{1}{g} = g \cdot \tfrac{1}{g}]$$

$$ghg^{-1} = h$$

$$ghg^{-1} \in H$$

∴ Hence,

H is Normal

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Theorem-2→ Intersection of any two Normal Subgroup of a group is again Normal Subgroup.

Proof→

Step 1→ let G be a group.

let H_1 and H_2 be a Normal Subgroup of G

⇒ we have to show that

$H_1 \cap H_2$ is again Normal.

Step 2⇒ Subgroup:-

∵ H_2 and H_2 is Subgroup.

$H_1 \cap H_2$ is also Subgroup

Normal

let $h \in H_1 \cap H_2$, $g \in G$

$h \in H_1$ and $h \in H_2$

↓ Normal ↓ Normal

$ghg^{-1} \in H_1$ and $ghg^{-1} \in H_2$

$\Rightarrow ghg^{-1} \in H_1 \cap H_2$

Hence $H_1 \cap H_2$ is Normal Subgroup. Ans

Theorem = 3⇒

let G be a group

H is Subgroup of G then

H is Normal Subgroup of G

(If and only Iff)

$xHx^{-1} = H, \forall x \in G$

Proof = Given that
G be a group
H is Subgroup of G

Part I → let H is Normal Subgroup of G — (1)
We have to show that
$xHx^{-1} = H, \forall x \in G$

Step I → $xHx^{-1} \subseteq H$
let, $xhx^{-1} \in xHx^{-1} \mid \forall h \in H$
∵ H is Normal [By Eq (1)]
∴ $xhx^{-1} \subseteq H$
Since ⇒ $xHx^{-1} \subseteq H$ — (2)

By replacing x by x^{-1} in Eq-(2)

$\Rightarrow x^{-1}H(x^{-1})^{-1} \subseteq H$
$\Rightarrow x^{-1}Hx \subseteq H$
Multiply both side by (x)

$\cancel{x}\cancel{x^{-1}}H\cancel{x}\cancel{x^{-1}} \subseteq xHx^{-1}$

$H \subseteq xHx^{-1}$ — (3)
By Eq (2) and Eq-(3)

$xHx^{-1} = H$

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II Converse Part

$xHx^{-1} = H$ —— (4)

$xHx^{-1} \in xHx^{-1}$

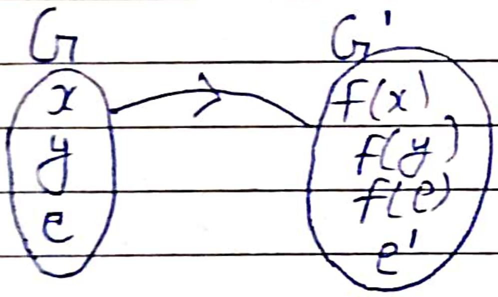
$xhx^{-1} \in H \rightarrow$ [By Eq-(4)]

$\therefore$ H is Normal Subgroup of G

II

★ Homomorphism :-

S= Let G and G' be a two group than $f : G \rightarrow G'$ is Called Homomorphism If $f(x \cdot y) = f(x) \cdot f(y) \; \forall \; x, y \in G$



Note→1] $f(G, +) \rightarrow (G', +)$

Ex $f(x+y) \rightarrow f(x) + f(y)$

$f(G, \cdot) \rightarrow (G', +)$ Note-2]

Ex:- $f(x \cdot y) \rightarrow f(x) + f(y)$

Note-3] $f(G, \cdot) \rightarrow (G', \cdot)$

Ex:-

$f(x \cdot y) \rightarrow f(x) \cdot f(y)$

Example:-

Q= $f : (G, +) \rightarrow (G', \cdot)$

S= $f(x) = e^{x} \; \forall \; x \in G$

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Homomorphism Step ①

S= Given that

$f(x)=e^x \quad \forall x \in G$ — ①

Step ② → $x, y \in G$

$f(x+y) = e^{x+y}$ → By Eq-①

$f(x+y) = e^x \cdot e^y$

$f(x+y) = f(x) \cdot f(y)$ → By Eq ① Ans

Q-2] $f:(G, \cdot) \to (G', +)$

$f(x) = \log x \quad \forall x \in G$

S= Homomorphism

Given that

$f(x) = \log x$

$f(x \cdot y) = \log(x \cdot y)$

$f(x \cdot y) = \log x + \log y \quad [\because \log(a \cdot b) = \log a + \log b]$

$f(x \cdot y) = f(x) + f(y)$ Proved

Q-3] $f:(G, +) \to (G', +)$

$f(x) = 2x$

S= $f(x+y) = 2x$

$f(x+y) = 2(x+y)$

$f(x+y) = 2x + 2y$

$f(x+y) = f(x) + f(y)$ Ans

Q=4] $f:(G,+)\to(G',\cdot)$

$f(x)=2^x \quad \forall x\in G$

$f(x+y)=2^x$

$f(x+y)=2^{x+y}$

$f(x+y)=2^x\cdot 2^y$

$f(x+y)=f(x)\cdot f(y)$ Ans

★ **Theorem**

If $f:G\to G'$ is homomorphism then

i] $f(e)=e'$

ii] $f(x^{-1})=[f(x)]^{-1}$

Proof G' Given

$f:G\to G'$

$f(x\cdot y)=f(x)\cdot f(y)$ — ①

Replacing y by e in Eq - ①

$f(x\cdot e)=f(x)\cdot f(e)$

$f(x)=f(x)f(e)$

$\frac{f(x)}{f(x)}=f(e)$

$e'=f(e)$

$\boxed{f(e)=e'}$

ii] replacing y by x^{-1} in Eq ①

$$f(x \cdot x^{-1}) = f(x).f(x^{-1})$$

$$f(e) = f(x).f(x^{-1})$$

$$\frac{f(e)}{f(x)} = f(x^{-1})$$

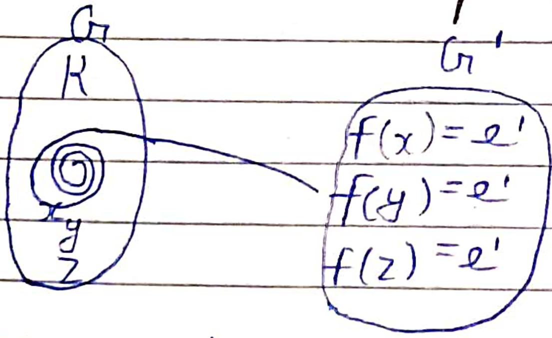
$$f(e)[f(x)]^{-1} = f(x^{-1})$$

$$e'[f(x)]^{-1} = f(x^{-1})$$

$$[f(x)]^{-1} = f(x^{-1})$$

$$\boxed{f(x^{-1}) = [f(x)]^{-1}}$$

★ <u>Kernal Homomorphism</u>



let G and G' be a two Group. $f : G \to G'$ is homomorphism Kernal of f is represented by K and it is denoted by :-

$$K = \{x \in G : f(x) = e'\}$$

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Theorem-① Kernal:-

Step-① Kernal K is Normal Subgroup

Proof-> Defination: let G and G' be a two group $f: G \to G'$ is homomorphism Kernal of f is represented by K and it is denoted by-

$$K = \{x \in G : f(x) = e'\}$$

Step② -> We have to show that K is Normal Subgroup.

Step③ -> Subgroup

i] $f(e) = e'$

$e \in K \to$ non Empty set

ii] $a, b \in K$

$f(a) = e'$ — ①

$f(b) = e'$ — ②

$\Rightarrow f(a \cdot b^{-1}) = f(a) \cdot f(b^{-1})$ | f is homomorphism |

$= f(a) [f(b)]^{-1}$ $\quad (\because f(x^{-1}) = [f(x)]^{-1})$

$= e' [e']^{-1}$

$f(a \cdot b^{-1}) = e'$

$ab^{-1} \in K$

K is Subgroup

ii] Normal

let $a \in K$, $x \in G$

$f(xax^{-1}) = f(x) f(a) f(x^{-1})$

$f(xax^{-1}) = f(x) e' f(x^{-1})$

$= f(x) \cdot [f(x)]^{-1}$ $\quad (\because f(x^{-1}) = [f(x)]^{-1})$

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$f(xnx^{-1}) = e'$

$xnx^{-1} \in K$

K is Normal Subgroup. Ans

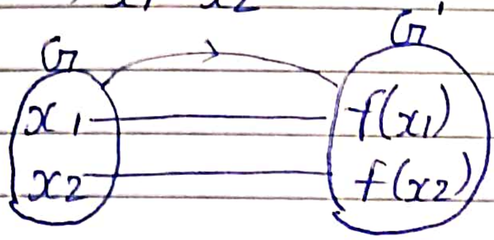
★ One One Mapping :-

:- Let G and G' be a two group and $f: G \to G'$ then f is called One-One.

If $x_1, x_2 \in G$

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$



Theorem :-

Let $f: G \to G'$ is homomorphism then f is one-One

If and Only Iff

$Ker(f) = e$

Given

$f: G \to G'$ is homomorphism —(1)

Part I

f is one- one :-

we have to show that

$Ker(f) = e$

$\therefore$ Let $x \in Ker(f)$

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$f(x) = e'$
$f(x) = f(e)$
$\therefore x = e$
$\boxed{Ker\, f = e}$ By Eq ①

Part II

$Ker(f) = e$ — ②

We have to show that f is One One

$x_1, x_2 \in G$
$f(x_1) = f(x_2)$
$f(x_1)\,[f(x_2)]^{-1} = e'$
$f(x_1) \cdot f(x_2^{-1}) = e'$

$f(x_1 x_2^{-1}) = e'$
$x_1 x_2^{-1} \in Ker(f)$
$x_1 x_2^{-1} = e$ | By Eq ②
$x_1 = x_2$

Hence
f is One One

★ **Onto** :-

$f : A \rightarrow B$

Domain Codomain
$x \in A$ $y \in B$

$f(x) = y$ → means y की Preimage x मे हुई तो onto होगा।

★ **Isomorphism** :-

Note → G and G' is Isomorphic → जब तीन Properties Isomorphism ho to G or G' जो है वो Isomorphic हो जाएगा।

Let G and G' be two group or $f: G \to G'$ than f is called Isomorphism If :-

i] One-One

ii] Onto

iii] Homomorphism

Unit = IV

* Linear Recurrence Relation with Constant Coefficient

* **Recurrence Relation** :- It is an Equation the recursively defines a sequence were the Next term is a function of previous term.

Ex = $S_n = \{2, 2^2, 2^3, ----\}$
$\quad\quad S_1 \; S_2 \; S_3 \; ----$

Finding S_{n-1} term | $S_1 = 2$

$S_n = 2S_{n-1}$

$S_2 = 2S_{2-1}$

$= 2S_1$

$= 2 \times 2$

$= 2^2$

Ex:- ② ->

$S = \{3, 7, 11, 15, 19, ---\}$

$S_1 = 3$

Q- $S_n = S_{n-1} + 4$

$n = 2$

$S_2 = S_{n-1} + 4$

$S_2 = S_{2-1} + 4$

$S_2 = S_1 + 4$

$S_2 = 3 + 4$

$S_2 = 7$

Recurrence Relation	Initial Condition
	$S_1 = 3$
$S_n = S_{n-1} + 4$	$n \geq 2$

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Ex-3] $S = \{\underbrace{1, 1}_{+}, 2, 3, 5, 8, 13 \ldots\}$

$S_n = S_{n-1} + S_{n-2}$

$n = 3$

$S_3 = S_{3-1} + S_{3-2}$

$S_3 = S_2 + S_1$

$S_3 = 1 + 1$

$S_3 = 2$

* Linear Recurrence Relation with Constant Cofficient

$\Rightarrow$

$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + C_3 a_{n-3} + \ldots + C_K a_{n-K} + \ldots = f(n)$

~~Where~~

Where $C_0, C_1, C_2 \ldots C_K$ is Constant Cofficient

Ex:-

① $2a_n + 3a_{n-1} = 3$

② $3a_n = -2a_{n-1}$

③ $13a_n - 2a_{n-1} + 3a_{n-2} = 0$

④ $a_n = a_{n-1} \cdot a_{n-2}$ / NOT Linear

⑤ $a_n \cdot a_{n-1} = a_n^2 - 2$ NOT Linear

* Order

$\Rightarrow$ It is the difference between highest and lowest

Ex:

1] $2a_{n} + 3a_{n-1} = 3$

$n - (n-1) \Rightarrow$ Highest - lowest

$= 1$

2] $3a_n = -2a_{n-1}$

$b = n-(n-1)$

$= 1$ Ans

3] $13a_n - 2a_{n-1} + 3a_{n-2} = 0$

$b = n-(n-2)$

$= 2$ Ans

* **Degree :**

It is define as the highest power of a_n

Note:- Degree of Linear Recurrence Relation is always 1

* **Method of Solving**

Linear Recurrence Relation with Constant Coefficient

$$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \cdots C_k a_{n-k} = f(n)$$

Homogeneous

$f(n) = 0$ होगा

non-Homogeneous

$f(n) \neq 0$ होगा

* **Step to Solve Homogeneous Equation**

①

put $a_n = r^n$

$a_{n-1} = r^{n-1}$

$a_{n-2} = r^{n-2}$

$\vdots \quad\quad \vdots$

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② Find an Eqⁿ in terms of γ this is called characteristics Eqⁿ or Auxiliary Equation.

③ Solve Charatesistics Equation and Find characteristics root

④ If Characteristics are different :-

i] root are $= \gamma_1, \gamma_2, \gamma_3$

$$a_n = b_1 \gamma_1^n + b_2 \gamma_2^n + b_3 \gamma_3^n$$

Ex:- $\gamma = 1, 2, 3$ then

$$a_n = b_1 1^n + b_2 2^n + b_3 3^n$$

ii] roots are same then

1] $\gamma = \gamma_1, \gamma_1$

$$a_n = (b_1 + n b_2) \gamma_1^n$$

2] $\gamma = \gamma_1 ; \gamma_1, \gamma_1$

$$a_n = (b_1 + n b_2 + n^2 b_3) \gamma_1^n$$

Ex:-

$\gamma = 2, 2, 2$

$$a_n = (b_1 + n b_2 + n^2 b_3) 2^n$$

iii] Two roots are same then different

$\gamma = \gamma_1, \gamma_1, \gamma_3, \gamma_4$

$$a_n = (b_1 + n b_2) \gamma_1^n + b_3 \gamma_3^n + b_4 \gamma_4^n$$

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Q=1 Solve $a_n = a_{n-1} + 2a_{n-2}$ with Initial Condition $n \geq 2$ $a_0 = 0$, $a_1 = 1$

S= Given, Step 1

$$a_n = a_{n-1} + 2a_{n-2}$$

Step-2:-

$$a_n - a_{n-1} - 2a_{n-2} = 0$$

put $a_n = r^n$

$$r^n - r^{n-1} - 2r^{n-2} = 0$$

$$r^n \left[1 - \frac{1}{r} - \frac{2}{r^2}\right] = 0$$

$$\Rightarrow r^n \left[\frac{r^2 - r - 2}{r^2}\right] = 0$$

$r^n \neq 0$

$$r^2 - r - 2 = 0$$

$$\Rightarrow r^2 + 1r - 2r - 2 = 0$$

$$\Rightarrow r(r+1) - 2(r+1)$$

$$(r+1)(r-2) = 0$$

$$r = -1, 2$$

$\because$ roots are different then

$$a_n = b_1(-1)^n + b_2 2^n \quad \text{—— (1)}$$

i] $a_0 = 0$

r^{-1} r^n $(x+1)(y+1) = 0$ $x-1=0$ $x=1$ $r^{n} = 0$ $y = -1$ $1, 2$ $r^2 - r - 2 = 0$

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By Eq ①

$a_0 = b_1(-1)^0 + b_2 2^0$

$0 = b_1 + b_2$ — ②

ii] $a_1 = 1$

$a_1 = b_1(-1)^1 + b_2 2^1$

$1 = -b_1 + 2b_2$ ---- ③

By Eq ② and ③

$b_1 + b_2 = 0$ — ④

$-b_1 + 2b_2 = 1$ — ⑤

$3b_2 = 1$

$\boxed{b_2 = \frac{1}{3}}$

Put the value of b_2 in Eq-④

$b_1 + \frac{1}{3} = 0$

$\boxed{b_1 = \frac{-1}{3}}$

Put In Eq ① we get,

$a_n = \left(\frac{-1}{3}\right)(-1)^n + \frac{1}{3} 2^n$ Ans

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r^2

Q-2] Solve $a_n = 4(a_{n-1} - a_{n-2})$ with Initial Condition

Condition $a_0 = 8,\ a_1 = 6,\ a_2 = 26$

→ Step 1

$a_n = 4a_{n-1} - 4a_{n-2}$

$a_n - 4a_{n-1} + 4a_{n-2} = 0$

Step 2

put $a_n = r^n$

$r^n - 4r^{n-1} + 4r^{n-2} = 0$

$\Rightarrow r^2 - 4r + 4 = 0$

$r^2 - 2r - 2r + 4 = 0$

$\Rightarrow r(r-2) - 2(r-2) = 0$

$\Rightarrow (r-2)(r-2) = 0$

$\boxed{r = 2, 2}$

$a_n = (b_1 + n b_2)2^n$ —— (*)

Step 3

i] $a_0 = 1$

By (*)

$a_0 = (b_1 + 0)2^0$

$1 = b_1$

$\boxed{b_1 = 1}$

ii] $a_1 = 1$

By (*)

$a_1 = (b_1 + b_2)2$

$1 = 2 + 2b_2$ $[\because b_1 = 1 = 1 \times 2 = 2]$

$r^r(r-2) = 2$

$a_n - 4a_{n-1} + a_{n-2} = 0$

$r^2 - 4r + 1$

$r^3 \quad r^2 \quad r$

r^3

4

$r^4 \quad r^3 \quad r$

$n(n-2)$

$n(n-3)$

$n^1 \quad 3$

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$-1 = 2b_2$

$\boxed{b_2 = -1/2}$

then put the value of b_1, b_2 in ($*$) :-

$$\boxed{a_n = \left(1 + \frac{n}{2}\right)2^n} \quad \text{Ans}$$

Q=3]

$a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with Initial ~~value~~ Condition.

$a_0 = 8, \ a_1 = 6, \ a_2 = 26$

S-

$a_n + a_{n-1} - 4a_{n-2} - 4a_{n-3} = 0$

put $a_n = r^n$

$r^n + r^{n-1} - 4r^{n-2} - 4r^{n-3} = 0$

$r^3 + r^2 - 4r - 4 = 0$

$r^2(r+1) - 4(r+1)$

$(r+1)(r^2-4) = 0$

$r = -1 \quad | \quad r^2 - 4 = 0$

$r^2 = 4$

$r = \sqrt{4}$

$r = \pm 2$

$r = -1, -2, 2$

$a_n = b_1(-1)^n + b_2(-2)^n + b_3(2)^n$ —— ($*$)

i] $a_0 = 8$

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By ⊛

$a_0 = b_1(-1)^0 + b_2(-2)^0 + b_3 2^0$

$8 = b_1 + b_2 + b_3$ — (i)

ii] By ⊛

$a_1 = b_1(-1)^1 + b_2(-2)^1 + b_3 2$

$6 = -b_1 - 2b_2 + 2b_3$ — (ii)

iii] $a_2 = 26$

By ⊛

$a_2 = b_1(-1)^2 + b_2(-2)^2 + b_3 2^2$

$26 = b_1 + 4b_2 + 4b_3$ — (iii)

By (i), (ii), (iii)

$b_1 = 2, b_2 = 5, b_3 = 1$

put the value of b_1, b_2, b_3 in eq ⊛

$$\boxed{a_n = 2(-1)^n + 5(-2)^n + 1(2)^n}$$

Practice Question

Q: $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 1, a_1 = -2, a_2 = -1$

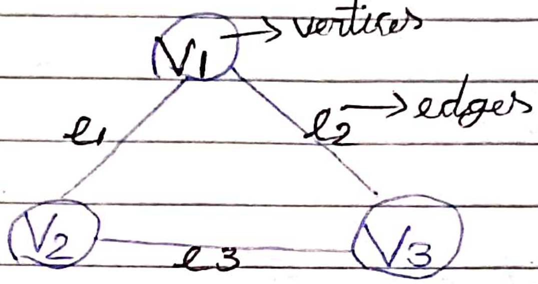
Ans: $(1 + 3n - 2n^2)(-1)^n$

Q: $a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 1, a_1 = 6$

A: $a_n = (1 + n)3^n$

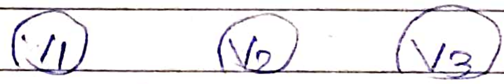
Unit-5
Graph Theory

A Graph is an ordered pair (V, E) where V is non-Empty infinite set of element called Vertices E is called Edges.

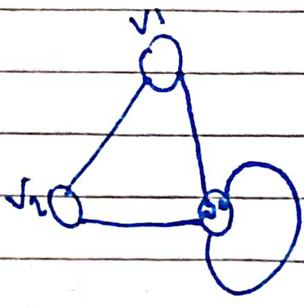


① **Trival Graph**:- Only One Vertex no Edges.

② **Null graph**:- No of Vertex No Edges



③ **Self loop**:- Edges having same vertex as both its End Vertices.



④ **Multi Edge**:- two or More Edge's having same End point

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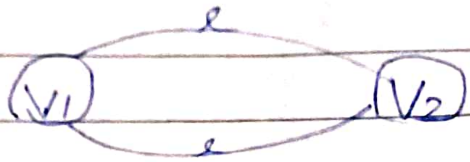
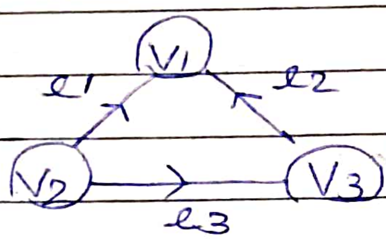


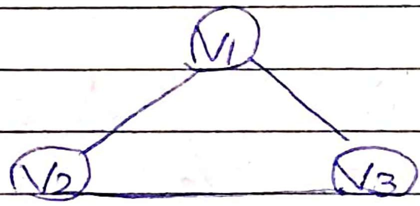
⑤ Directed Graph :-

A= Direction of Edges in the Graph



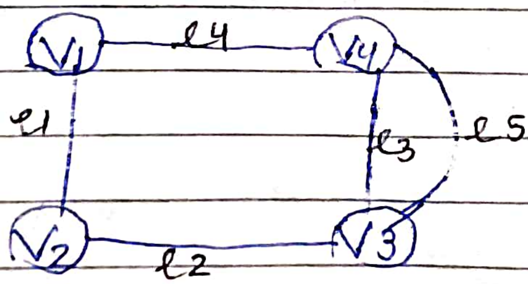
⑥ Simple Graph :-

A= A graph does not contain any self loop or Multi Edge



⑦ Multi Graph :-

A= A Graph having Multiple Edges is called Multigraph



⑧ Degree of vertex → Degree of vertex V is Equal to the No of Edges which are incident (on) on V

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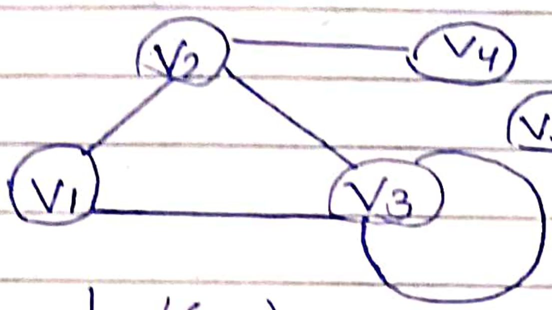
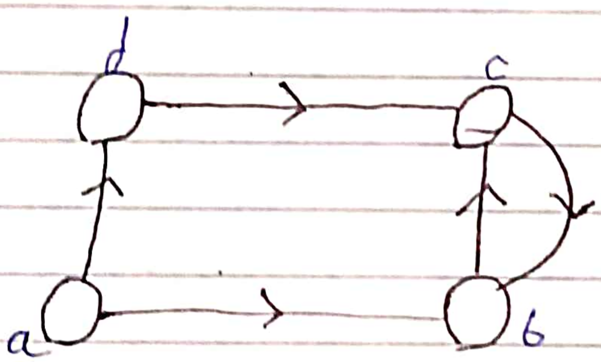
⇒ Self loop Counted twice



$d(v_1) = 2$ | $d(v_4) = 1$
$d(v_2) = 3$ | $d(v_5) = 0$
$d(v_3) = 4$

II)



Vertices	In degree	out degree
a	0	2
b	2	1
c	2	1
d	1	1

* Note:- Isolated Vertex
⇒ A Vertex having Degree (zero)

Pendent Vertex
⇒ A Vertex having Degree (one)

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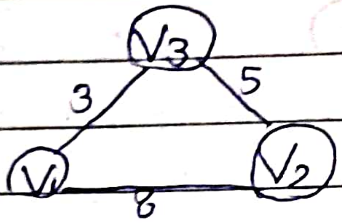
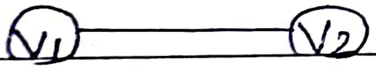
⑨ Weighted Graph :- A graph in which some weight is Assigned to every edge of Graph.

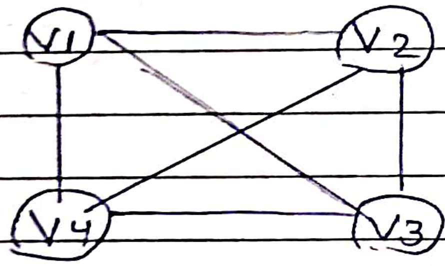


⑩ Complete Graph -> A Graph in which each vertex Connected To every other vertex.

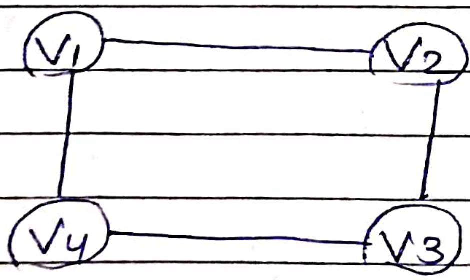
Ex:-



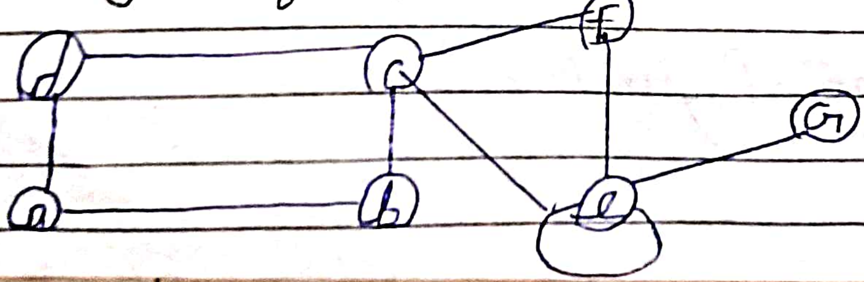
2]



⑪ Regular Graph :- Every vertex have same degree



Q- Find degree of Graph :



1] $d(a)=2$
2] $d(b)=2$
3] $d(c)=4$
4] $d(d)=2$
5] $d(e)=5$
6] $d(f)=2$
7] $d(G)=1$

* Adjacency Matrix

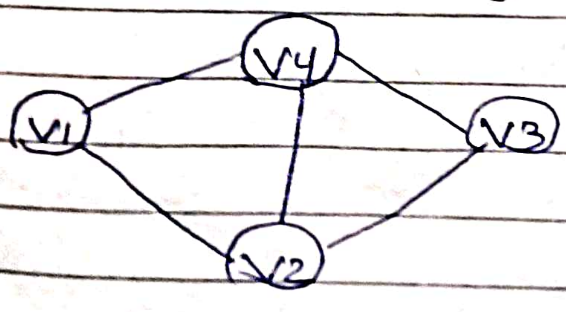
$$\begin{array}{cc} & \begin{matrix} V_1 & V_2 & V_3 & V_4 & \cdots \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ \vdots \end{matrix} & \left[\begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \qquad\qquad\qquad \right] \end{array}$$

Connected = 1
Not Conn = 0

* Incidency Matrix

$$\begin{array}{cc} & \begin{matrix} e_1 & e_2 & e_3 & \cdots \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ \vdots \end{matrix} & \left[\begin{matrix} \\ \\ \\ \\ \end{matrix} \qquad\qquad\qquad \right] \end{array}$$

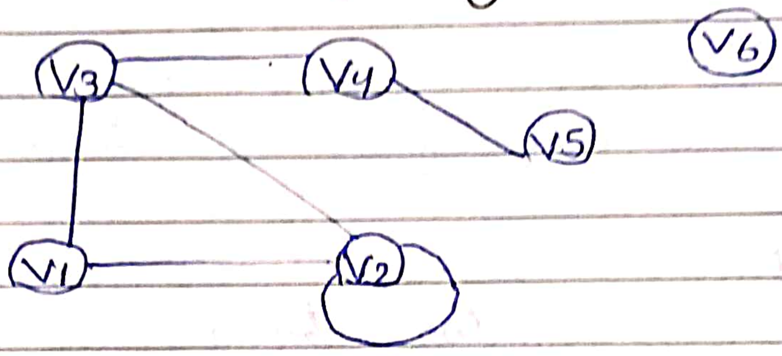
Ex:- Q - Find the adjacency Matrix.



$$D = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_2 \\ \hline V_1 & 0 & 1 & 0 & 1 \\ V_2 & 1 & 0 & 1 & 1 \\ V_3 & 0 & 1 & 0 & 1 \\ V_4 & 1 & 1 & 1 & 0 \end{array}$$

Ans

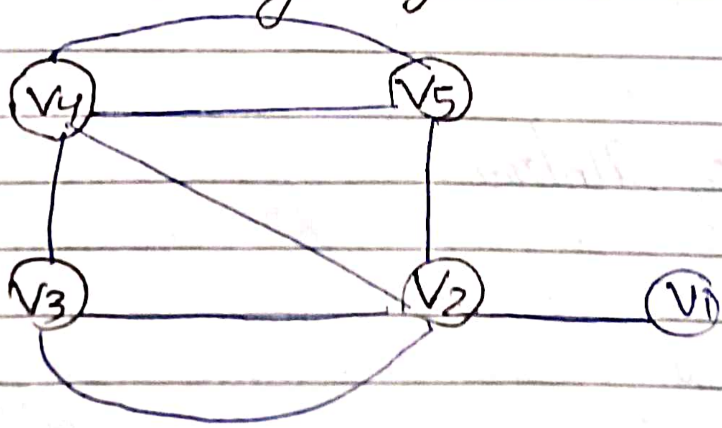
Q-2] Find the adjacency Matrix



$$\begin{array}{c|cccccc} & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\ \hline V_1 & 0 & 1 & 1 & 0 & 0 & 0 \\ V_2 & 1 & 1 & 1 & 0 & 0 & 0 \\ V_3 & 1 & 1 & 0 & 1 & 0 & 0 \\ V_4 & 0 & 0 & 1 & 0 & 1 & 0 \\ V_5 & 0 & 0 & 0 & 1 & 0 & 0 \\ V_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

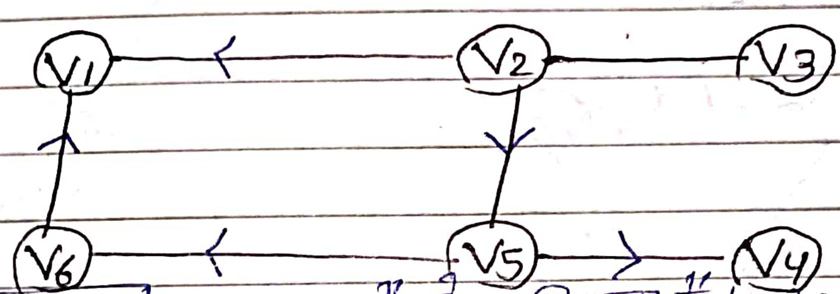
Ans

Q-3] Find the adjacency matrix



$$
\begin{array}{c|ccccc}
 & V_1 & V_2 & V_3 & V_4 & V_5 \\
\hline
V_1 & 1 & 1 & 0 & 0 & 0 \\
V_2 & 1 & 0 & 2 & 0 & 1 \\
V_3 & 0 & 2 & 0 & 1 & 0 \\
V_4 & 0 & 0 & 1 & 0 & 2 \\
V_5 & 0 & 1 & 0 & 2 & 1
\end{array}
$$

Q= Find the adjacency Matrix

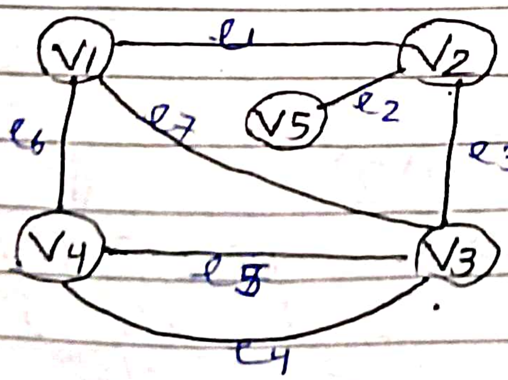


$\boxed{Out = 1}$ → जा रहा है तो 1 लिखना है direction

b=

$$
\begin{array}{c|cccccc}
 & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\
\hline
V_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 0 & 1 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 1 & 0 & 1 & 0 & 1 \\
V_6 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Q= Find the Incidence Matrix

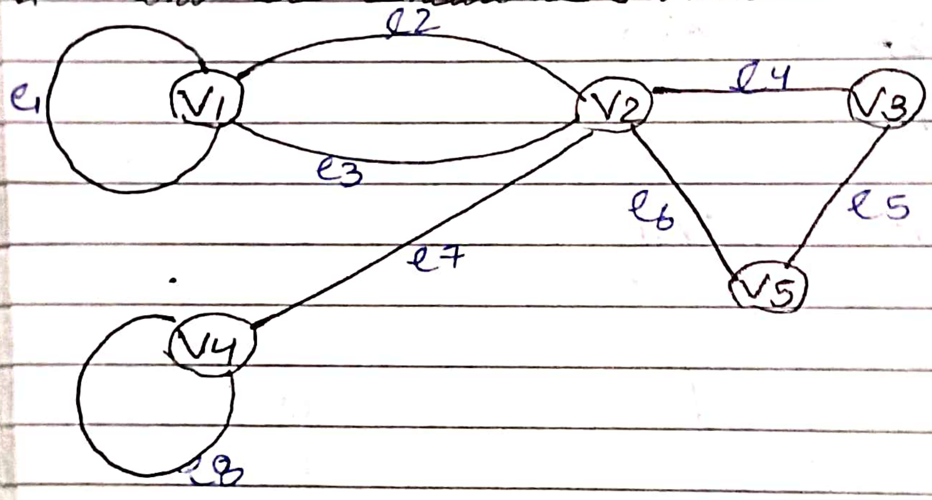


$$
\begin{array}{c|ccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\
\hline
 & & & & & & & \\
\end{array}
$$

$$
D = \begin{array}{c}V_1\\V_2\\V_3\\V_4\\V_5\end{array}\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

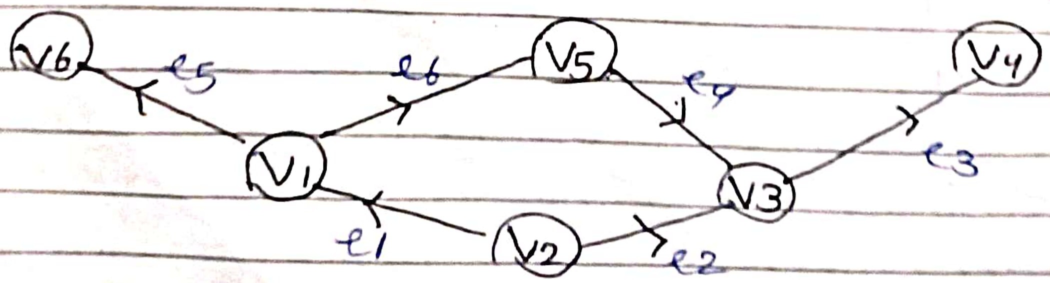
Ans

Q= Find the Incidence Matrix



$$
\begin{array}{c|cccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\
\hline
V_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
V_2 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
V_3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
V_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{array}
$$

Q= Find the Incidence Matrix.

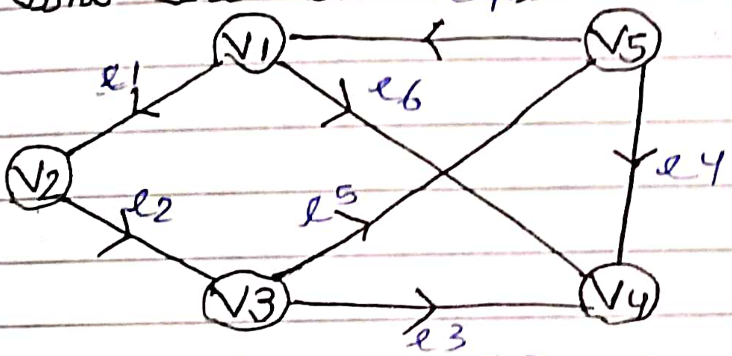


Note: In = -1
Out = 1

S =	e1	e2	e3	e4	e5	e6
V1	-1	0	0	0	1	1
V2	1	1	0	0	0	0
V3	0	-1	1	-1	0	0
V4	0	0	-1	0	0	0
V5	0	0	0	1	0	-1
V6	0	0	0	0	-1	0

Ans

Q. Find the incidence Matrix.



	e1	e2	e3	e4	e5	e6	e7
V1	1	0	0	0	0	1	-1
V2	-1	1	0	0	0	0	0
V3	0	-1	0	0	1	0	0
V4	0	0	-1	-1	0	-1	0
V5	0	0	0	1	-1	0	1

Ans

★ **Eulerian Graph**

Eulerian Path :- 0 or 2 Vertices of odd degree

⇒ Every Edges of graph Appears exactly Once in the path.

Eulerian Circuit :-

⇒ Each Vertices must be degree

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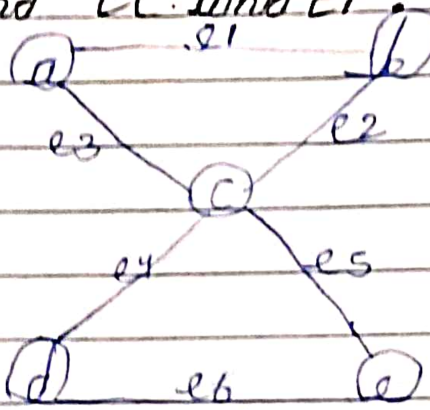
=> the circuit which contains Every Edge of the graph exactly once

Ex:- Eulerian Graph:

Q1 Find EC and EP.



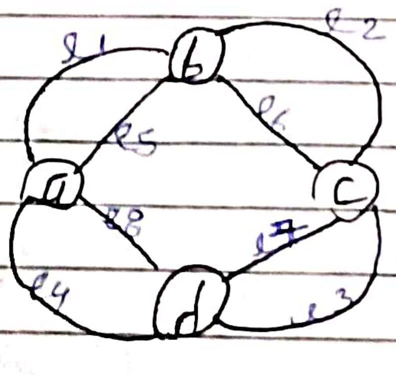
Step① Definition

Step②

E.P = a e1 b e2 c e4 d e6 e e5 c e3 a Ans

E.C = Same Ans

Q=2] Check wether the graph has Euler circuit Euler path, Justify



Step 1 -> Defination

Step 2 :-

E.P a e1 b e2 c e3 d e4 a e5 b e6 c e7 d e8 a Ans

E.C = Same Ans

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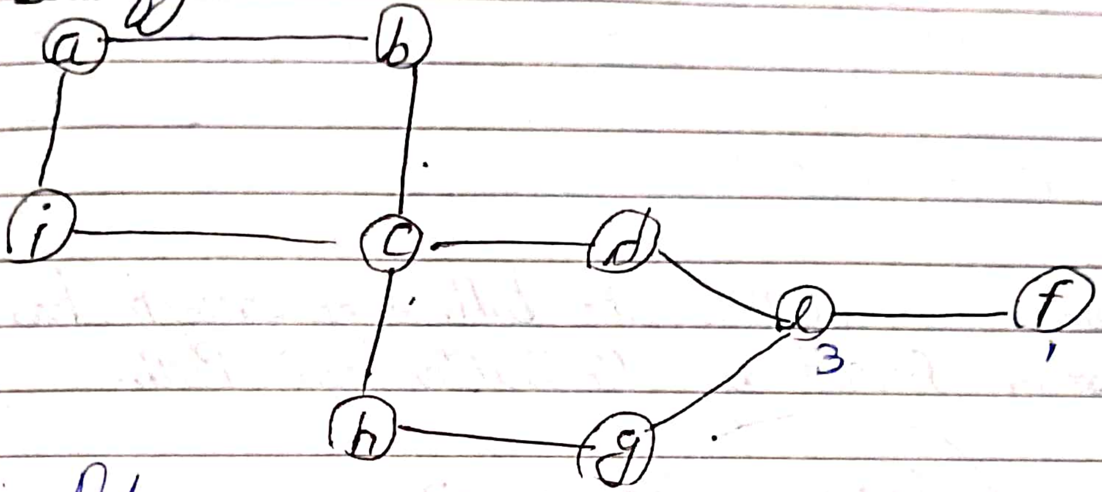
Q-3] Check whether the graph has Euler Circuit, Euler path. Justify.



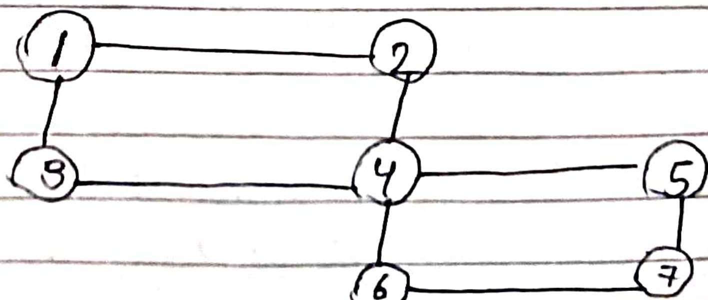
S= Step 1 Def

EP = f e g h c b a i c d e
EC = Does not Exist

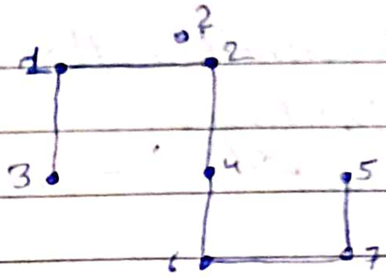
Imp
★ Hamiltonian Graph:-
S= Hamiltonian path:- Every of graph appers Exactly once in the path.

Hamitonian circuit :- The circuit which contains Every vertex of the graph exactly once and Ending point is same.

Q= Determine whether the following graph has a Hamilton circuit or Hamiltonian path.
S=
Step-1 Definition

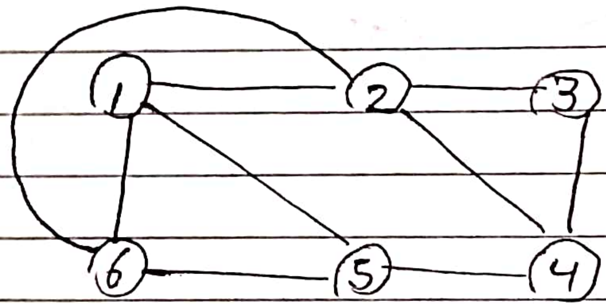


Step 1 Definition
Step 2 → H.P
= 3124675 Ans

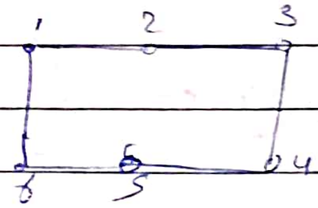


H.C = Does not Exist Ans

Q-27 Determine whether the following graph has a Hamiltonian Circuit or Hamiltonian Path.

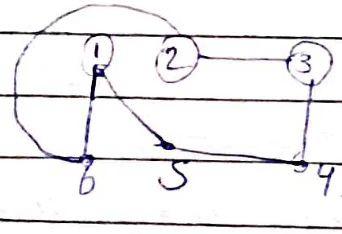


S = Step 1 → Defination
Step 2 - H.P = 234516



1) Ans / 2 Ans

H.C = 1234561 / 2345162

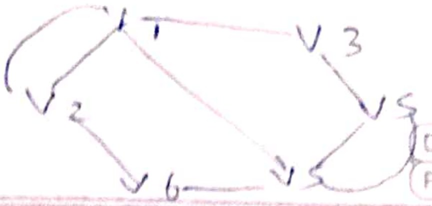


★ Handshaking Lemma :-

$\sum_{i=1}^{n} d(v_i) = \cancel{2\times e}, 2\times e$ multiply — degree निकालनी है किसी graph कि तो 2×e – edges कर दो।

Result ⇒ Max degree of any Vertex in a Simple graph with n Vertices is $(n-1)$

$16 = 2\times 6$
$e = \frac{2}{8}$



Ex: Q-1) Show that the Maximum no of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

S= Maximum degree of any vertex in a simple graph with n vertices is $(n-1)$

By Hand-Shaking Lemma

$$\sum_{i=1}^{n} d(v_i) = 2\times e$$

$$d(v_1)+d(v_2)+d(v_3)+\cdots+ = 2\times e$$

$$= (n-1)+(n-1)+(n-1)+\cdots = 2\times e$$

$$n(n-1) = 2\times e$$

$$\boxed{e = \frac{n(n-1)}{2}}$$ Ans proved

Q- Determine the no of edges in a grap with 6 vertex, 2 of degree 4 and 4 of degree 2, Draw such graphs,

S= According to Question:

$d(v_1)=4$ $\quad$ $d(v_3)=2$
$d(v_2)=4$ $\quad$ $d(v_4)=2$
$d(v_5)=2$
$d(v_6)=2$

By Hand Shaking Lemma

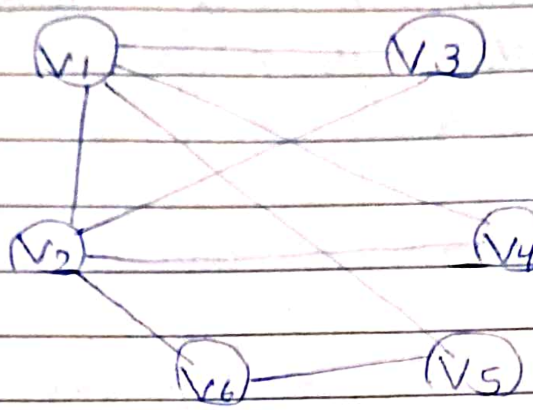
$$\sum_{i=1}^{6} d(v_i) = 2\times e$$

$$= d(v_1)+(d v_2)+d(v_3)+d(v_4)+d(v_5)+d(v_6) = 2\times e$$

$$= 4+4+2+2+2+2 = 2\times e$$

$$16 = 2\times e \rightarrow \boxed{e = 8}$$ Ans

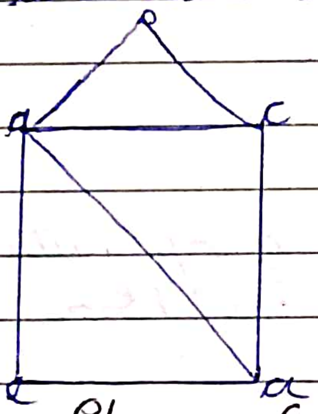
Graph:-



★ Planar Graph:-

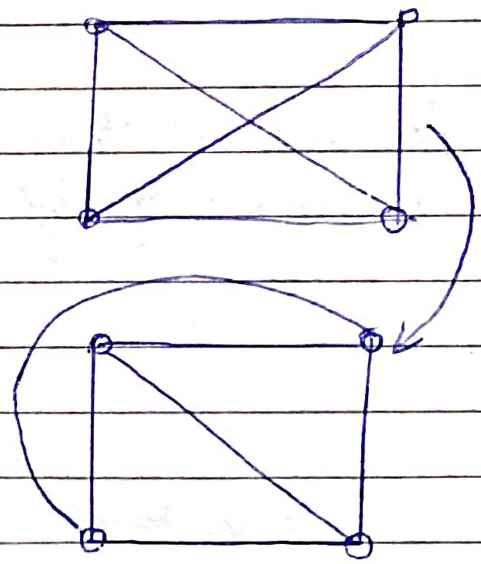
A= A graph is said to be planar if it can be drawn as a plane such a way that No Edges cross one another

Ex:-



Planar-G

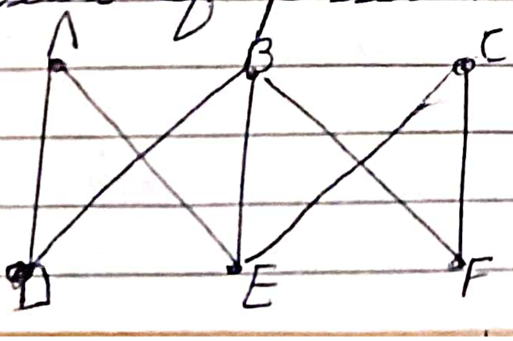
Not a P.G



Convert a Planar Graph

Ex: Q= Draw a Planar represented of graph given below if possible.

i]



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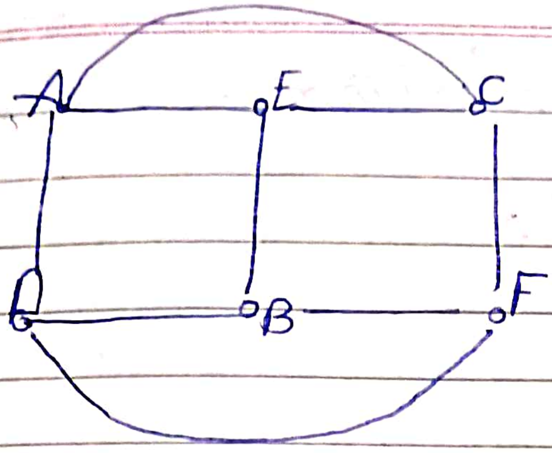
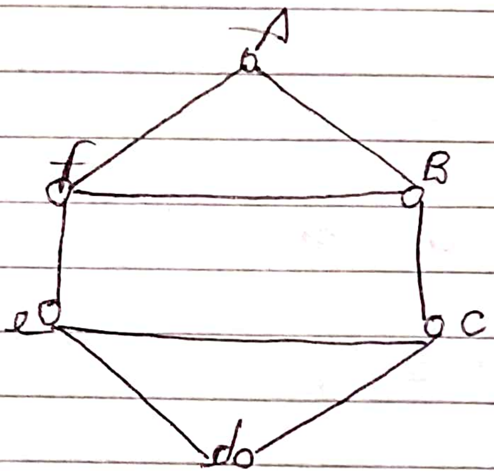
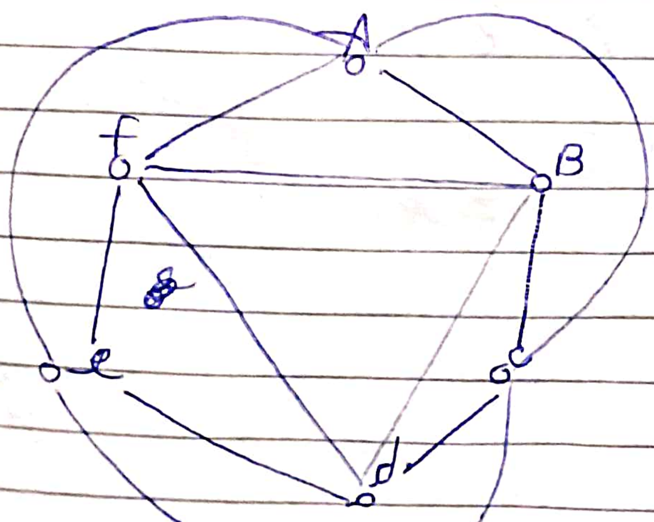
b=



Q-27 Draw a Plancos represented of group given below if possible

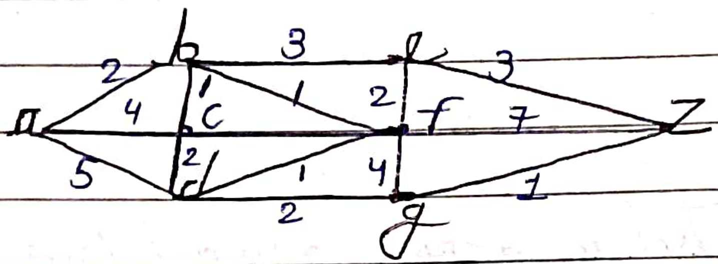


b=

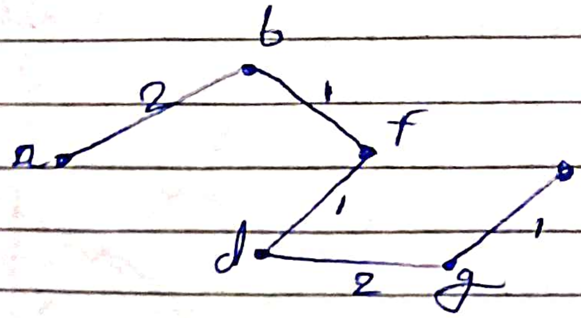


★ Imp **Dijkstra's Shortest path Algorithm**

Q- Use Dijkstra's Shortest path :-



b =	a	b	c	d	e	f	g	z
	$\boxed{0}$	2	4	5	∞	∞	∞	∞
Fix हो जाएगा								
	$\boxed{0}$	$\boxed{2}$	3	5	5	3	∞	∞
	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	5	5	3	∞	∞
	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	4 $\boxed{\cancel{3}}$	5	$\boxed{3}$	7	10
	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	5	$\boxed{3}$	6	10
	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{3}$	6	8
	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{3}$	$\boxed{6}$	7

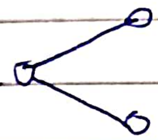


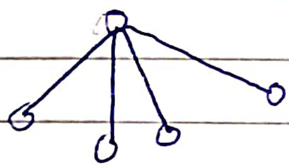
* Tree

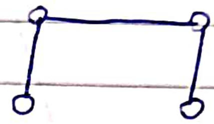
⇒ A tree is a Simple Connected Graph without any Circuit

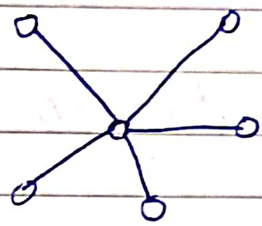
i] 

ii] 

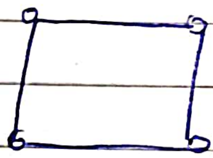
iii] 

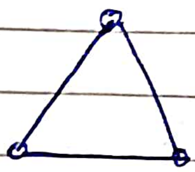
iv] 

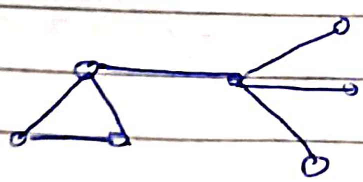
v] 

vi] 

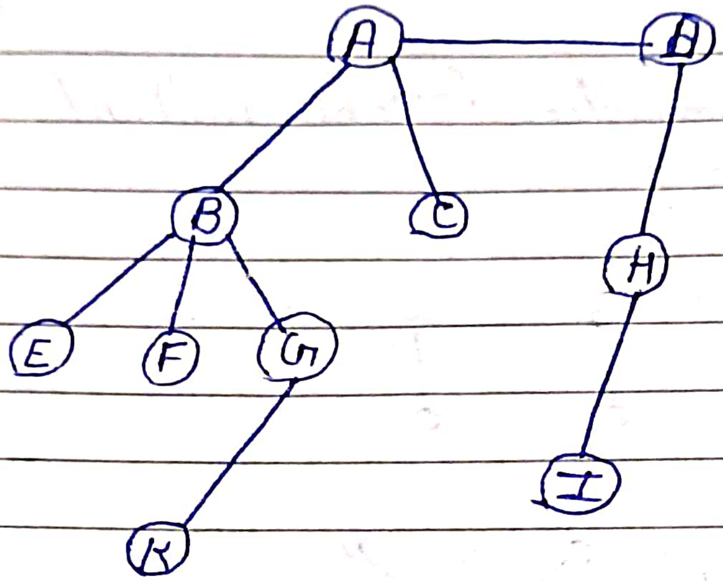
Not Tree :-

vii]  Not Tree

viii]  Not Tree

ix]  Not Tree

* Types of Tree :-



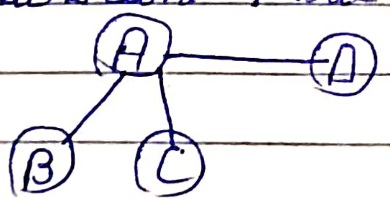
① Root of the tree = A, D

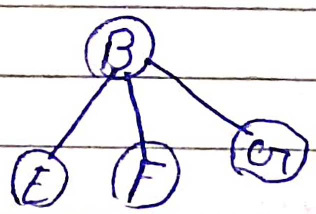
② Degree of Node

$d(A)=3, \ d(B)=3, \ d(C)=0$
$d(D)=2$

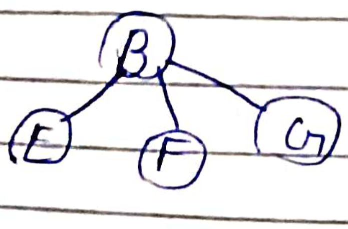
③ Path of the Tree
$A \to B \to G \to K$

④ Parrent Node :-





⑤ Child Node :-



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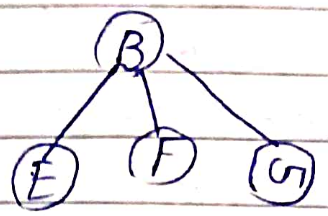
(6) Siblings Node:



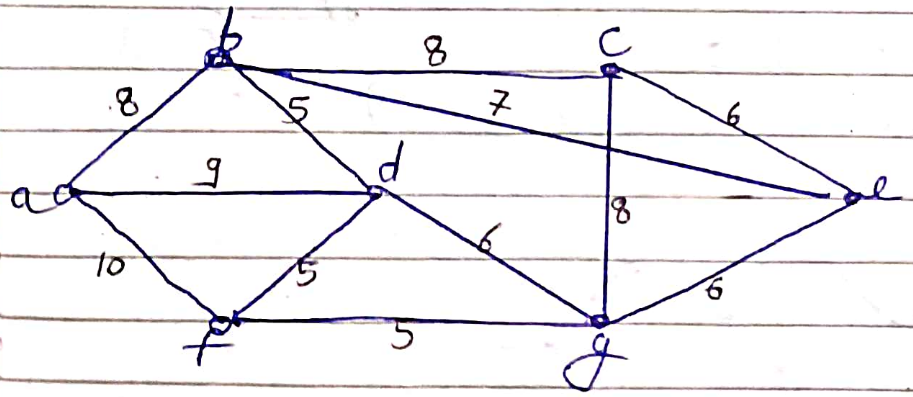
(7) Leaf Node:

=> Node degree Zero :-
(K), (I), (E), (F), (C)

(8) Internal Node :- Root or leaf ke whose degree atleast 1,
Internal Node are present between Root Node and Leaf Node.
(B) (G) (H) (I)

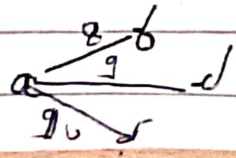
IMP
★ Minimum Spanning Tree

Q= Find the Minimum Spanning Tree for the following using prims algorithm



b= Step 1 :-
A= Select Vertex a
$MST = \{(a), ( )\}$

Step = 2

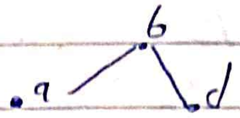


→ minimum distance leni hai

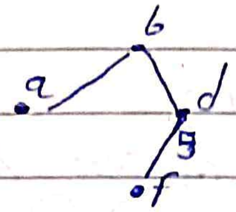
Date :
P. No. :

Vertex a Connected to b
MS.T = {(a, b) (e1)}

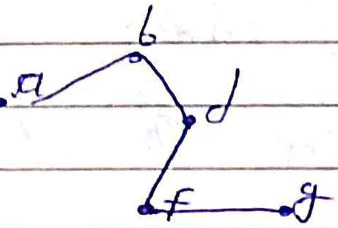
Step 3 Vertex b Connected to d
MST = {(a, b, d) (e1, e2)}



Step-4]
Vertex d Connected to f
MST {(a, b, d, f) (e1, e2, e3)}

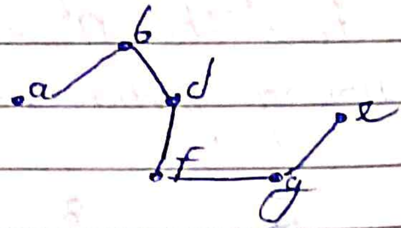


Step 5] :-
vertex f Connected to g
MST {(a, b, d, f, g) (e1, e2, e3, e4)}

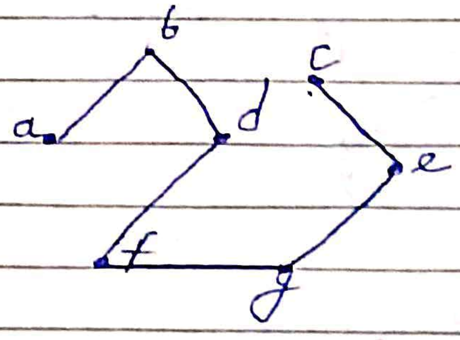


Step 6]
Vertex g Connected to e

MST {a, b, d, f, g, e) (e1 e2 e3 e4 e5)



Step 7] Vertex e Connected to c
MST {a, b, d, f, g, e, c) (e1 e2 e3 e4 e5 e6)



Order se direct निकल जाएगा.
कितने order की equation होगी

Unit = 4 Extra Question

Date:
P. No.

* Linear Recurrence Relation :-

$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} = f(n)$

$f(n) = 0$

* Non - Homogeneous

$f(n) \neq 0$

* Non- Homogeneous Recurrence Relation

b= General Solution=

$$a_n = \underbrace{a_n^{(h)}}_{\text{homo. Sol}^n} + \underbrace{a_n^{P}}_{\text{Particular Sol}^n}$$

i] $f(n) = \alpha$ (Constant)

1] let $a_n = A$

2] Put $a_n = a_{n-1} = a_{n-2} = \cdots = A$ in given Recurrence relation

3] Find A

4] $a_n^P = A$

Ex:-

Solve $a_{n-2} - 5a_{n-1} + 6a_n = 2$ with initial Condition $a_0 = 1$, $a_1 = -1$.

b= put $a_n = r^n$

$\Rightarrow r^{n-2} - 5r^{n-1} + 6r^2$ 2

$\Rightarrow r^2 - 5r + 6$

$\Rightarrow r = 2, 3$

i] $a_n^h = b_1 2^n + b_2 3^n$

ii] a_n^p

$\Rightarrow A - 5A + 6A = 2$

$-2A = 2$

$\boxed{A = 1}$

$a_n^p = 1$

$G.S = a_n^h + a_n^p$

$a_n = b_1 2^n + b_2 3^n + 1$ —— $\star$

i] $a_0 = 1$

put in $\star$

$a_0 = b_1 2^0 + b_2 3^0 + 1$

$1 = b_1 + b_2 + 1$

$\boxed{b_1 + b_2 = 0}$ —— ①

ii] $a_1 = 1$

$a_1 = b_1 2^1 + b_2 3^1 + 1$

$-1 = 2b_1 + 3b_2 + 1$

$2b_1 + 3b_2 = -2$ —— ②

By ① and ②

$b_1 = 2$, b_2 2,

$b_1 + b_2 = 0$

$-2b_1 + 3b_2 = -2$

$-b_1 - 2b_2 = 2$

→ elimination

se and

[illegible]

Data :
P. No :

HW= $a_n - 6a_{n-1} + 8a_{n-2} = 3$

$$b_1 + b_2 = 0$$
$$-2b_1 \mp 3b_2 = \mp 2$$
$$b_1 - 2b_2 = 2$$

$b_1 - 2b_2 =$